2022 HiMCM Problem A: The Need for Bees (and not just for honey)
Team 12525

Summary

Bees are critical to the world's ecosystem. Besides providing humanity with honey, they also pollinate crops – a process essential for keeping plants genetically diverse and for agricultural production. In recent years, the population of honey bees has decreased due to various factors. This phenomenon has gained enough recognition that it has been given the name Colony Collapse Disorder. While there are many suspected reasons for Colony Collapse Disorder, the primary influencing factors for this decline remain unclear. To assess probable causes, initial models were created to illustrate the population of a bee colony and the locations of colonies relative to each other.

First, a model was developed to calculate the population of a colony of honeybees on a certain day. To do this, the colony population was split into 5 subpopulations based on the honey bee life cycle: larvae, pupae, male drones, female workers working in the hive, and female workers foraging outside of the hive. The number of eggs was also considered directly for the larva population, but they were not counted towards the colony population. Once this was determined, a general population model was created reflecting only the colony itself; external factors were not accounted for in this particular model. All of these subpopulations were modeled using similar techniques - the interdependence and sequencing of the subpopulations are reflected as the model is factored on the life cycle of the bee. After that, external factors were incorporated into the model; these being more specific to the subspecies. For example, the risk of disease in the colony was accounted for as a limiting factor.

Second, the factors that contributed to the model were analyzed to determine which most strongly affected the population. A few of the factors that were considered include birth rate, temperature, and risk of illness. To do this, a Python Jupyter Notebook was leveraged to create a colony population model. This Python notebook allowed for a dynamic model; changing each variable and observing which caused the greatest change in population.

A model was developed that calculated the number of hives needed to pollinate a 20-acre area of land based on specific aspects of the population model. This area was assumed to be in the shape of a square. The range of land that one hive could pollinate was assumed to be circular. The radius of this area was determined by the population of the hive. The radius also considered the crops that required pollination in the 20-acre area. Each crop requires a different number of bees to pollinate and may vary in crop density. Certain aspects of a given crop were accounted for in our model.

Additionally, an infographic was constructed with the aim of educating the general population about the various factors that affect the bee populations and the number of hives needed to pollinate a given square plot of pollinated crops to support farmers in ensuring maximum crop productivity.

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1 Problem Introduction

Problem Statement

For some people, when they think of bees, they think about how they sting you and perhaps how they would argue against a multi-billion dollar corporation in court about honey. Due to various factors such as diseases, the use of pesticides, habitat loss, and climate change, the population of bees has been declining each year (Pollinators at a Crossroads, n.d.). However, that begs the question: why should people care about keeping honey bees alive?

Honeybees and a few other key animals are an essential part of our ecosystem. Aside from making honey and asking people if they like jazz, bees support the crucial pollination process of plants that feed the ever-growing population (Brown & Paxton, 2009). Because of the global decrease in the bee population, a term called Colony Collapse Disorder (CCD) was created in 2007 to define the sudden decline of bee populations around the world. Possible factors contributing to CCD include viruses, pesticides, predators, habitat destruction, and environmental conditions (US EPA, 2013).

The goal of the problem is to deliver multiple models that would meet the requirements provided by HiMCM Problem A. The requirements are to develop a model to determine the population size of a honeybee colony, determine which factors have the greatest impact on the population of a honeybee colony, another model to predict the number of hives needed to support pollination of 20 acres of land, and a one-page infographic that would provide the information found in this paper for a non-scientific audience.

Initial Research of the Problem

The problem provides a small amount of information as a starting point for the required model. Additional data was gathered in order to find other factors that may influence the population of a bee colony. From this research, a model was to be created that would be affected by factors such as temperature, age of bees, the survival rates of bees in different stages, the energy consumption of bees, and other variables that would impact the population.

Learning about how a hive functions is essential to constructing a model. In order to create a model, a general understanding of the components of a hive and the purpose of all the workers was necessary. Being able to obtain information about why colonies started and were destroyed was also useful.

2 Assumptions

1. The queen is the only bee that lays eggs. (worker bees can lay eggs as the queen bee's pheromones become weaker (*Honeybee Colony*, n.d.))
2. The colony population is counted as the number of total larvae, pupae, and adult bees in said colony. Unhatched eggs are not counted towards the colony's total population.
3. Assume that worker bees do not switch their role until they are at the age to do so (Mortensen & Ellis, 2019).
4. Since the queen bee only mates once in her lifetime, drones that die due to mating are negligible and are not counted in the population model (*Bee Life Stages*, 2006; *Mechanics of Honey Bee Mating*, n.d.).
5. Assume that any bees that become infected with a disease will not survive.

6. The 20-acre area for part 7 is assumed to be a square.
7. The area that a hive can pollinate for part 7 is assumed to be circular.

3 Constructing the General Model

3.1 Bee Life Cycle

The evolution of a colony's population is assumed dependent on the number of bees in a given life cycle stage: larva, pupa, and adult. Fertilized eggs become female worker bees, while unfertilized eggs become male drone bees (University of California San Diego). The role of a worker bee shifts given its age: younger worker bees (referenced throughout this paper as 'workers') stay within the nest, while more mature worker (referenced as 'foragers') bees collect pollen and nectar necessary for food and any other necessary resources. A honeybee's necessities, survival, and associated dangers are dependent on life cycle stages and are thus considered separately. As a result, the total bee population P_{total}, can be written in terms of the population of each respective life cycle stage (P_{stage}), where *stage* represents the life cycle stage of the bee.

$$P_{total} = P_{larva} + P_{pupa} + P_{drone} + P_{worker} + P_{forager}$$

Within each stage, the number of honeybees in a given day can be separated into three distinct categories: number of new stage bees per day (N), the population of bees in a given stage at the beginning of the day (C), the number of bee deaths per day (D), and the number of bees which progress onto the next stage of development (E) - which is applicable for all stages except drones and workers. In order to model this effectively, the population is measured at the end of a given day.

$$P_{stage} = N + C - D - E$$

In the following sections, the parameterizations for each term of the above equation are derived given factors related to the life cycle stage.

3.2 General Population Progression

The measurement of the honeybee population necessitates the consideration of honeybee lifespan and the number of honeybee deaths per day given a specific stage. Due to the large number of factors that may affect a bee's lifespan, a survival rate (S_d) is expressed as the likelihood that a bee will survive on a given day (d). To calculate the final population of a group born on the same day (P_f) within d days and given an initial population (P_i), the product of all daily survival rates must be taken (see Appendix 10.2.1). As the model of the beehive measures population at the end of a given day, the product's upper bound is inclusive of d.

With this equation, the number of bees that reach the next lifecycle stage can be derived in terms of the new stage population ($N[d - v]$) v days prior to the current day (d), where v a constant denoting the development period of the given stage.

$$E = N[d - v] \cdot \prod_{k=d-v}^{d} S_k$$

The life cycle stage daily death quantity (D) can also be expressed in terms of the number of current stage members not encompassed by the daily survival rate. This value takes into account the population loss detailed within the E equation and does not consider the number of bees that transitioned to the current stage.

$$D = (C) \cdot (1 - S_d)$$

A more robust parameterization of the population of a bee life cycle stage given date (P_{stage}[d]) can now be constructed. All terms can also be expressed in terms of the date. Note that the number of new members per stage is equal to the previous stage's amount of members lost due to the transition ($E_{prev}[d]$). The number of current members ($C[d]$) of the given stage is also equivalent to the stage population of the previous day ($P_{stage}[d - 1]$). Parenthesis are placed around terms for differentiation.

$$P_{stage}[d] = N + C - D - E$$

$$P_{stage}[d] = (E_{prev}[d]) + (P_{stage}[d - 1]) - (P_{stage}[d - 1] \cdot (1 - S_d)) - E[d]$$

$$P_{stage}[d] = (E_{prev}[d]) + (P_{stage}[d - 1] \cdot S_d) - E[d]$$

The above equation can be applied for the larva and pupa stages of the bee. The drone and forager stages of a bee's life serve as the final developmental step for bees and thus do not consider stage transformation quantity, indicating that the final term for this equation can be neglected. The population function for the forager bee population is given below.

$$P_{stage} = E_{prev}[d] + P_{stage}[d - 1] \cdot S_d$$

However, more specific considerations regarding bee population must be taken into account for the worker and drone stages of bee development, since their emergence rates are dependent on egg-laying behavior, and an equation for these bees is developed in the following section.

3.3 Egg-laying Behavior

3.3.1 Hatch Rate

The population growth of the hive is heavily dependent on the total number of eggs laid by the singular colony queen bee (*Bee Hive Hierarchy and Activities*, n.d.; *Honeybee Colony*, n.d.; University of California, Division of Agriculture and Natural Resources, n.d.). Studies show that a productive queen can lay thousands of eggs a day (*LESSON 3*, 2011; Rueppell et al., 2007). These eggs–which are deposited by the queen into a honeycomb cell called a brood–are subsequently sheltered by worker bees for warmth during a process called "brood rearing" (University of California, Division of Agriculture and Natural Resources, n.d.). It must be noted that the number of eggs hatched is equivalent to the number of new honeybee larvae per day. Similar to the equations detailed within Section 3.2, the equation for the number of eggs hatched (H_{egg}) can be expressed in terms of the egg-laying rate of the queen bee (l_{egg}), the egg incubation period (v) and egg survival rate (S_d).

$$H_{egg} = N_{larva} = l_{egg}[d - v] \cdot \prod_{k=d-v}^{d} S_k$$

3.3.2 Egg Fertilization and Job Specification

The role of a mature honeybee within the nest is determined by whether its egg was fertilized by the queen: fertilized eggs become female worker bees, while unfertilized eggs grow into male drones (*Honeybee Colony*, n.d.). The fertilization rate of eggs is primarily dependent on the choice of the queen, but may also change based on hive population, season, queen age, and so on (*How Do Bees Choose the Next Queen? | Wonderopolis*, n.d.). In general, the fertilization ratio can be expressed as f. Throughout the larval and pupal stages of development, the identity of a bee as a worker or drone is assumed to not substantially affect the needs nor the survival rate. As a result, the fertilization ratio can be applied to the number of surviving pupae (E_{pupa}) to determine the number of drones (N_{drone}) and the amount of workers (N_{worker}) which emerge on a given day.

$$N_{worker} = E_{pupa} \cdot f$$

$$N_{drones} = E_{pupa} \cdot (1 - f)$$

Now, updated population equations for all life cycle stages can be derived. Expanded equations can be found in Appendix 10.2.2.

4 Model Analysis

4.1 Testing Method

The above population equations were written in a Python Jupyter Notebook. The egg laying rate, egg fertilization rate, stage population, and survival rates for each bee life cycle stage were regarded as adjustable variable values considered constant throughout the population model. Future considerations may include the dependence of these values on environmental factors, which are briefly touched on in Section 5. The hive model begins after an initial stage of hive progress and each stage's population is evenly distributed throughout its respective developmental cycle. For example, the stage population within a specific day of development (P_{day}) is expressed in terms of the total stage population (P_{stage}) divided by the development period (v_{stage}).

$$P_{day} = \frac{P_{stage}}{v_{stage}}$$

The population of each respective stage – as detailed in the above equations – was derived by day and stored within an array value. When a function required a previous population value, the designated index within the array was called. The code returns the array of total hive population progression over a date range as well as the population on the specified date.

4.2 Population Simulation

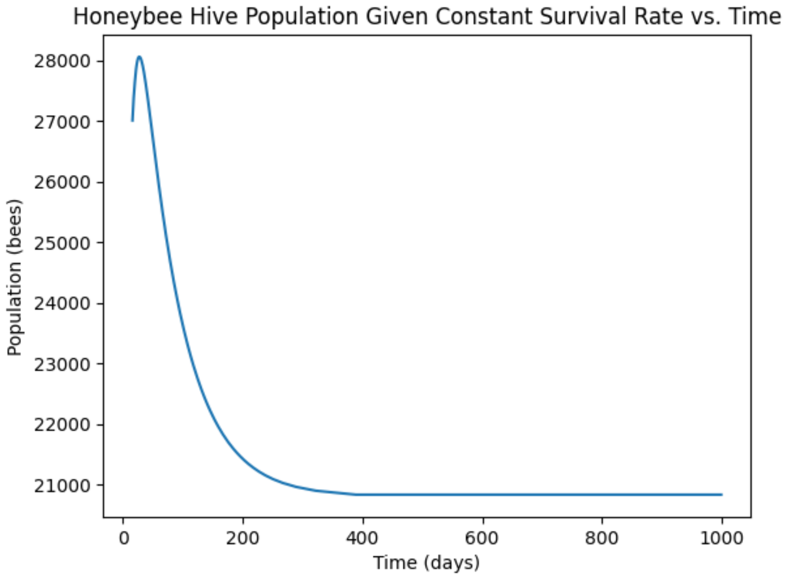
To assess model functionality, values extrapolated from real-world research were placed into the given Python variables. According to the findings of Torres, David J., et al. 2015, the approximate daily survival for an observed bee population is detailed below.

S_{egg}	S_{larva}	S_{pupa}	S_{drone}	$S_{forager}$
.94	.917	.985	.985	.9

While the survival rate of worker bees was not detailed, it was assumed to be equivalent to S_{pupa} and S_{drone}, as all three reside within the hive. Additionally, the average egg-laying rate of the queen bee was expressed as 2000 eggs per day, and the fertilization rate was approximately 0.99; there is one drone for every 99 female workers and forager bees. The initial bee population per life cycle stage was heuristically estimated within the chart below.

P_{larva}	P_{pupa}	P_{worker}	P_{drone}	$P_{forager}$
11000	24000	20000	300	10000

Given these values, the population of the hive was modeled as a function of time in days and displayed within a chart. The population was measured over 1000 days to visualize the gradual evolution of the hive over time.



4.3 Discussion

The above graph displays a hive's bee population over 1000 days. The population of the beehive initially increases as eggs are inputted into the population. However, the population reaches its maximum at the 27th day of development. This likely results from the number of deaths of each respective stage becoming greater than the number of bee hatches. To suppress the steep peak in the bee population, a more realistic distribution of the initial bee population should be considered. After approximately 300 days, the bee population asymptotically approaches 20836 bees. This is resultant from the gradual balance of bee births and bee deaths. This value is considered reasonable, as the survival rate remains consistent and the bee population remains within the estimated ideal 20,000 to 60,000 population range. It should be noted that the change in season over large durations in the real world significantly affects bee behavior and population.

4.4 Sensitivity Analysis

As this population model depends on a myriad of different factors, it is important to consider how alterations within each factor may independently affect the total population of the colony. Each independent factor was multiplied by 0.95 and 1.05 respectively to indicate the effect of a smaller and larger value on the population. All larger survival rates were capped at 1, meaning that bees do not die within this stage. The newly updated quantity was subsequently applied to the simulation constructed in Section 5.2 and the total population was measured at 1000 days. The new population was divided by the previously extrapolated

1000-day population to achieve a ratio denoting the sensitivity of the model with regard to that variable. The sensitivity ratios of all considered independent variables of the model are listed below.

	l_{egg}	l_f	s_{egg}	s_{larva}	s_{pupa}	s_{drone}	s_{worker}	$s_{forager}$
x0.95	0.952	1.189	0.858	0.919	0.968	0.964	0.989	0.793
x1.05	1.051	0.811	1.159	1.058	1.004	$1.304*10^{14}$	0.995	1.546

The most substantial factor on population when it was decreased was the forager survival rate, as the most adult bees become foragers within their lifetime; the more substantial the death rate, the steeper the overall population decline.

Additionally, the most substantial factor on population when it was increased was the drone survival rate, as no number of bees are lost due to developmental transformation, and the survival factor maps to 1. As no bees die during this stage, the amount of bees continues to increase linearly over the duration of population measurement.

5 Life Span and Survival Rate

In this model, the honeybee's survival rate is assumed to be dependent on the stage of life and evaluated daily. However, the factors which affect survival rate are generalized into three primary categories: the risk of health defects caused by exposure to pathogens, pesticides, etc. (s), and temperature behavior. Each of the following are detailed below.

5.1 Temperature and Season Change

Temperature and Season change is an important aspect of a bee's population. Temperature affects the efficiency of the bees. With Temperature affecting how worker bees and forager bees act, the overall efficiency of a hive can change based on the current season. A season change mainly affects the temperature in the model. The models and equations below include temperature as a factor.

5.2 Risk of Sickness

According to scientists, there are at least 20 different viruses that affect bee populations and several strains of bacteria (EPA, 2022). These illnesses may be caused by a myriad of factors, but are considered to be resultant from a universal contamination risk of the honey within the hive. The model considers the mortality risk associated with infection to be dependent on the developmental stage with a constant risk of infection within the r hive; recently hatched larvae are the most susceptible to pathogens. Within the larval stage, the mortality rate decreases linearly from 1 (100% mortality risk from contracting an illness) to a baseline mortality rate of c, as it reaches the pupa stage. The mortality rate for illness is constant within the pupa and drone stages of the bug.

$$m_{larva}[d] = \frac{c-1}{v} \cdot d + 1$$

$$m_{pupa}[d] = m_{drone}[d] = c$$

6 Number of Hives for 20 Acres

6.1 Simple Model

From the assumption that the 20-acre (81,000 square meters) plot of land was square, a model to describe the number of hives needed to support the crops on the given parcel of land was constructed. The model depends on the number of bees needed to support each crop (x), the number of plants per square meter (d), the number of foraging bees per hive (p), and the range of the bees (r).

To begin with, the maximum area that a single bee hive could cover while ignoring the plants (A) was calculated.

$$A = \pi r^2$$

From there the number of plants within range for a given hive was found to be $\pi r^2 d$. Next, the number of plants that would be supportable given the population of foraging bees per hive (p) and the number of bees needed to support each plant (x) was found to be $\frac{p}{x}$. Now, since two expressions were obtained that both represented the number of plants that could be supported by a hive, they could be set equal to one another in order to isolate r.

$$\frac{p}{x} = \pi r^2 d$$

$$r^2 = \frac{p}{\pi x d}$$

$$r = \sqrt{\frac{p}{\pi x d}}$$

It was known that the radius (r) could not be more than 20,000 meters as that is the maximum distance that a bee would fly from their hive. However, the bees may also need to travel to the corners not covered by their circular area. Thus:

$$r\sqrt{2} \leq 20000$$

$$r \leq \frac{20000}{\sqrt{2}}$$

$$r \leq \frac{20000\sqrt{2}}{2}$$

$$r \leq 10000\sqrt{2} \text{ meters}$$

From there, the side lengths of the plot of land were calculated to be $90\sqrt{10}$ as the total area was given as 81,000 square meters and $90\sqrt{10}$ is the square root of 81,000. It was then decided that in order to find the number of hives needed for the square plot of land given the radius that each hive could cover, the length of one side of the plot would need to be divided by 2r as the length covered by each hive would be 2r on each side. Once that value was determined to be $\frac{90\sqrt{10}}{2r}$, it could then be squared to find the rough number of hives needed.

$$\left(\frac{45\sqrt{10}}{r}\right)^2$$

We then substituted in the previously calculated value of r into the expression above.

$$\left(\frac{45\sqrt{10}}{\sqrt{\frac{p}{\pi x d}}}\right)^2$$

The expression above represents the number of hives needed to support the given plot of land. It is advisable to round the value up if the result is not an integer value to ensure that the hives could have complete coverage over the given area.

The steps above do simplify the scenario somewhat by ignoring the space between the areas covered by the bees.

6.2 Improved Model

In order to account for the gaps between the area covered by each hive, a second version of the model to determine the number of hives needed to cover the given area was constructed. The process started the same as the process for the last model. However in order to calculate *r*, the area of the overlap between the different bee hive areas had to be added before the area that a hive could cover based on the number of plants within an area (the left side of the equation below) could be set to the area the hive could cover based on the number of bees needed to support the number of plants in a given area (the right side of the equation below).

$$\pi r^2 d + (\pi - 2)r^2 = \frac{p}{x}$$

From there, *r* was isolated.

$$r^2(d\pi - (\pi - 2)) = \frac{p}{x}$$

$$r^2 = \frac{p}{x(d\pi - (\pi - 2))}$$

$$r = \sqrt{\frac{p}{x(d\pi - (\pi - 2))}}$$

Since it is known from the previous work shown in section 7.1 that the number of hives is equivalent to the square root of the area over 2*r* squared, the following equation was created.

$$\left(\frac{\sqrt{81{,}000}}{2 * \sqrt{\frac{p}{x(d\pi - (\pi - 2))}}}\right)^2 = \text{The number of hives needed to pollinate 81,000 square meters}$$

It is again advisable to round the number of hives up if the result is not an integer as having less than the required amount of bees would cause the crops in the 20 acres to not receive enough treatment from bees.

6.2.1 Almond Trees

This model was applied to different scenarios. For example, in this section, the model for the number of hives was applied to a scenario where the crops in the given area were almond trees. According to *One Bee for Every 20 Nuts* by Aaron Smith, there are on average 6,000 almonds per tree, 125 trees per acre, and 1 bee for every 20 almonds. There are also 4046.86 square meters per acre.

The number of bees per tree was calculated to be: $\frac{6000\ almonds}{1\ tree} * \frac{1\ bee}{20\ almonds} = \frac{300\ bees}{tree}$. Next the number of trees per square meter was calculated to be:

$\frac{1\,Acre}{4046.8\,square\,meters} * \frac{125\,trees}{1\,Acre} \approx 0.0308881454\,trees/square\,meter$. If the foraging population of each hive was assumed to be the average population of a hive (60,000 bees), then the number of hives needed to support the given plot of land with almond trees would be 126. This value was calculated by plugging the values that were just discussed into $(\frac{\sqrt{81{,}000}}{2*\sqrt{\frac{p}{x(d\pi-(\pi-2))}}})^2$. If preferred, the population can be left as a variable in the final expression. If so, the expression representing the number of hives needed in this scenario would be $(\frac{\sqrt{81{,}000}}{2*\sqrt{\frac{p}{(300)((125/4046.86)\pi-(\pi-2))}}})^2$ where p represents the population of foraging bees.

7 Conclusions

7.1 Discussion of the General Population Model

The model related to population of a colony takes into account many factors that would heavily impact a colony's population. The model uses variables that change every day such as temperature and population at the current moment.

The strengths of this model would include how many factors are taken into account. The real world has a large amount of variables and is why models in general are sometimes inaccurate. This is why the model created having more variables gets the model closer to a realistic prediction. The model challenges how close we are able to get to what a real colony's population would look like.

What could also be considered a strength of the model can also be seen as a weakness? Although our model is strong as it contains many variables, the number of variables present creates a complex model that requires a lot of data. The model contains variables such as initial population, egg-laying rate, and other variables. The problem with these variables is that some of them are hard to obtain. On a large scale, it would be hard to measure the number of larvae, meaning that the numbers would have to be gained through predictions and other data. The number of variables is significant as it requires people to gather more data for the model.

If given more time, the model could have ways of using estimates for some variables, and include more variables to make it more accurate. It may sound like a contradiction to speak of using estimates as variables, but these estimates could be used as a strength since somebody who would like to model a colony's population can spend less time gathering data for each hive that they wish to use. Another thing is being able to simplify the model by using more substitution. Because of the time constraint, the model contains variables that could be turned into different variables and perhaps combined in order to create a model with less complexity while retaining the same output.

7.2 Discussion of the Number of Hives Model

The model related to the number of hives required to sustain a field uses variables that are easily identifiable. The model was constructed using variables such as the population of a beehive and the amount of pollination that the plants in the area require. This model, like the previous one about population, has some strengths and weaknesses.

The model does not generalize what kind of crops are used inside the 20-acre area. The model requires data on the type of crops, but in return, a more accurate representation of how many bees are required is shown. Measuring the individual number of bees in stages of their life can be difficult, but measuring the population the number of bees that leave the hive is much easier. Although it still is measuring the individual number of bees inside of a stage, this does not require looking into the hive at all and an estimate can be used for a number like this. The model uses the total population of the colony and as a result, requires a reasonable amount of data for a problem to be solved.

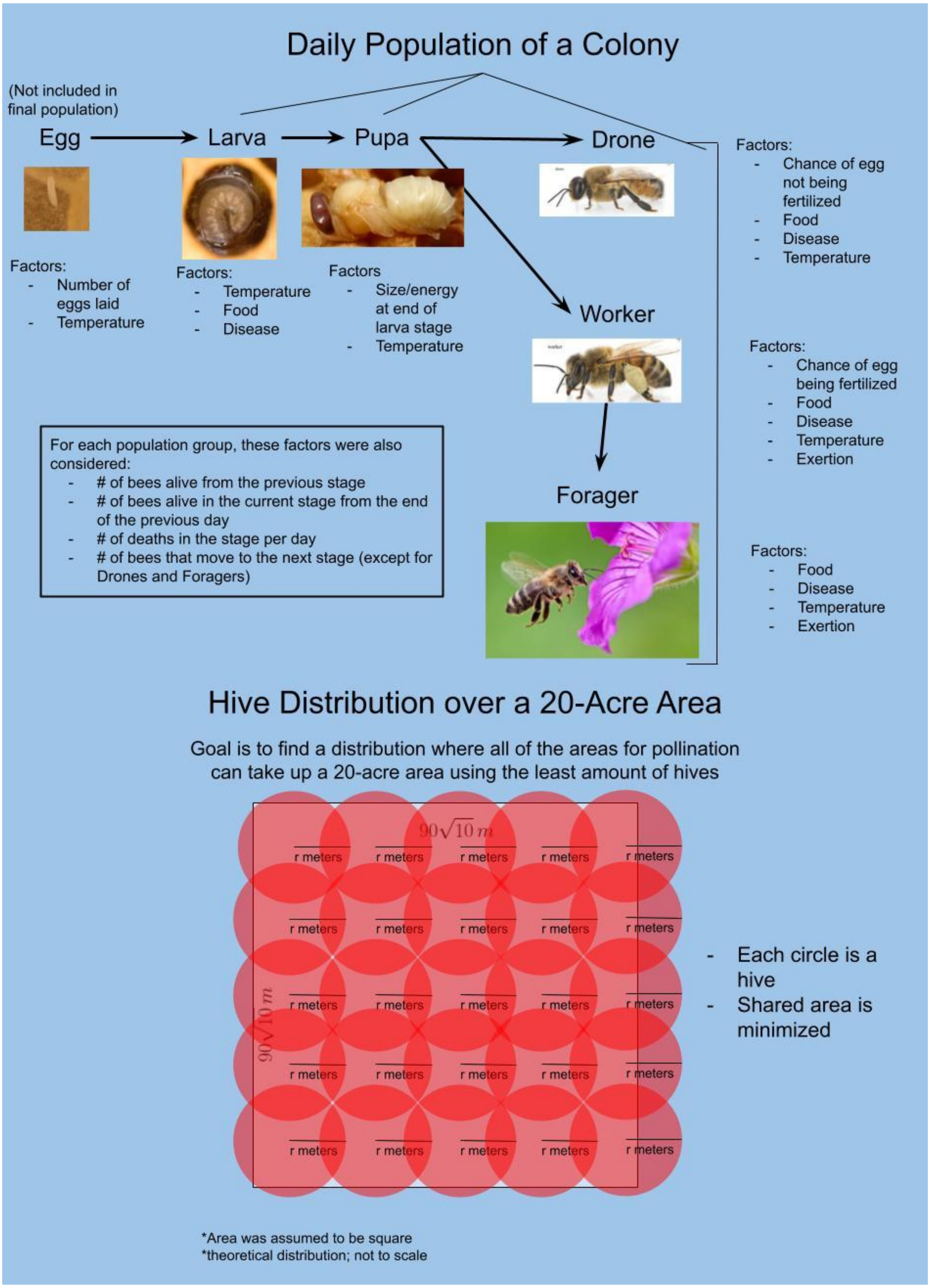
The model shows potential weaknesses that are related to the behavior of bees. If resources are low and bees from different hives are collecting their nectar and pollen, there is a likely chance that they will fight for the resources (Adriana, 2021). They will defend what they claim as their resources. Another weakness is that the bees' movement is described as a circle when in reality the bees can move in irregular patterns that would miss crops in certain spots.

With more time some more problems can be addressed. It is nearly impossible to model the movement of a bee accurately, so a circle was the best that was possible. The spacing between the hives can be improved in order to prevent conflict between hives. With how close each hive can be to each other (in terms of proximity) it is inevitable that they will meet, however, it is also possible to find a good spot that would allow for more proper spacing that would reduce the interactions between two colonies.

Both models show a weakness that can be solved with more time. This weakness is that both models do not consider most causes of how a model can fail. Both models do not consider situations such as pesticides, predators, and habitat loss. Most of these situations are the main contributors to Colony Collapse Disorder. Adding more variables to the model (and considering how different locations effect the model) would be a huge factor in most of these situations.

Overall, the models are able to represent data gathered from any amount of sources and predict the population of the colony over time. It uses both obscure data (data that would require and general data which allows for a more accurate prediction of the real world. The model has trouble considering events that occur rarely or unpredictable events, but this is a problem that will most likely happen with all mathematical models.

8 Infographic



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10 Appendices

Appendix 10.1 Method Analysis Code

10.1.1 Population Function

```
import numpy as np
```

```
INCUBATION_DURATION = 3
LARVA_DURATION=6
PUPA_DURATION=12
WORKER_DURATION=17
egg_laying_rate = 2000
egg_fertilization_rate = .99
egg_survival_rate = .94
larva_survival_rate = .917
pupa_survival_rate=.985
drone_survival_rate=.985
worker_survival_rate=.985
forager_survival_rate=.9
larva_pop_i = 11004
pupa_pop_i = 23640
worker_pop_i = 20000
drone_pop_i = 301
forager_pop_i = 10000
def death_term(date, current_stage_population: list, survival_rate: float) -> int:
    return np.round(current_stage_population[date-1]*survival_rate)
def num_eggs_hatched(date: int, eggs_per_day: list, egg_survival_rate, incubation_durtion=3) -> int:
    return np.round(eggs_per_day*egg_survival_rate**incubation_durtion)
def num_transformed(stage_population:list, stage_duration:int, d:int, stage_survivalRate:float) -> int:
    if d-stage_duration<0: return 0
    else:
        return np.round(stage_population[d-1]*stage_survivalRate**stage_duration)
def get_stage_population(date: int, prev_transformed: int, stage_population: list, stage_survival_rate: float,
stage_duration)->int:
    stage_survival_rate=np.min([1.0, stage_survival_rate])
    transformed = num_transformed(stage_population, stage_duration, date, stage_survival_rate)
    return [prev_transformed+death_term(date, stage_population, stage_survival_rate)-transformed,
transformed]
def getRoleNum(e_f_rate: float, new_adults: int) -> list:
    workers = np.round(e_f_rate * new_adults)
    return [workers, new_adults-workers]
def dispersePop(tot_stage_pop, stage_duration, max_duration=17):
    pop_evo = np.empty(max_duration)
    for i in range(max_duration):
        if i > max_duration - stage_duration -1 and i <= max_duration:
            pop_evo[i] = np.round(tot_stage_pop/stage_duration)
        else:
            pop_evo[i] = 0
    return pop_evo
def getPop(df, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate,
pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate):
    larva_pop = dispersePop(larva_pop_i, LARVA_DURATION)
    pupa_pop = dispersePop(pupa_pop_i, PUPA_DURATION)
```

```
    worker_pop = dispersePop(worker_pop_i, WORKER_DURATION)
    drone_pop = dispersePop(larva_pop_i, 1)
    forager_pop = dispersePop(larva_pop_i, 1)
    total_population = dispersePop(larva_pop[16] + pupa_pop[16] +worker_pop[16]+
forager_pop[16]+drone_pop[16], 1)
    for date in range(17, df):
        eggs_hatched=num_eggs_hatched(date, egg_laying_rate, egg_survival_rate)
        p_larva=get_stage_population(date, eggs_hatched, larva_pop, larva_survival_rate,
LARVA_DURATION)
        larva_pop = np.append(larva_pop, (p_larva[0]))
        p_pupa=get_stage_population(date, p_larva[1], pupa_pop, pupa_survival_rate, PUPA_DURATION)
        pupa_pop = np.append(pupa_pop, (p_pupa[0]))
        p_worker=get_stage_population(date, getRoleNum(egg_fertilization_rate, p_pupa[1])[0], worker_pop,
worker_survival_rate, WORKER_DURATION)
        worker_pop = np.append(worker_pop, (p_worker[0]))
        p_forager=get_stage_population_final(date, p_worker[1], forager_pop, forager_survival_rate)
        forager_pop = np.append(forager_pop, (p_forager))
        p_drone=get_stage_population_final(date, getRoleNum(egg_fertilization_rate, p_pupa[1])[1],
drone_pop, drone_survival_rate)
        drone_pop = np.append(drone_pop, (p_drone))
        total_population=np.append(total_population,
p_larva[0]+p_worker[0]+p_pupa[0]+p_drone+p_forager)
    return [total_population, total_population[df-1]]
general_total_population = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate,
larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)
```

10.1.2 Population Graph Display Code

```
import matplotlib.pyplot as plt
x = np.arange(16, 1000, 1)
plt.figure()
plt.plot(x, general_total_population[0][x])
plt.title('Honeybee Hive Population Given Constant Survival Rate vs. Time')
plt.xlabel('Time (days)')
plt.ylabel('Population (bees)')
plt.show()
```

10.1.3 Sensitivity Analysis Code

```
def get_ratio(newPop, general_total_population=general_total_population[1]):
    return newPop/general_total_population

less_egg_laying_rate = getPop(1000, .9*egg_laying_rate, egg_fertilization_rate, egg_survival_rate,
larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate,
forager_survival_rate)[1]
more_egg_laying_rate = getPop(1000, 1.1*egg_laying_rate, egg_fertilization_rate, egg_survival_rate,
larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate,
forager_survival_rate)[1]
```

less_egg_fertilization = getPop(1000, egg_laying_rate, .9*egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]
more_egg_fertilization = getPop(1000, egg_laying_rate, 1.1*egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]

less_egg_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, .9* egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]
more_egg_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, 1.1*egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]

less_larva_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, .9*larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]
more_larva_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, 1.1*larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]

less_pupa_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, .9*pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]
more_pupa_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, 1.1*pupa_survival_rate, drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]

less_drone_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, .9*drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]
more_drone_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, 1.1*drone_survival_rate, worker_survival_rate, forager_survival_rate)[1]

less_worker_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, .9*worker_survival_rate, forager_survival_rate)[1]
more_worker_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, 1.1*worker_survival_rate, forager_survival_rate)[1]

less_forager_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, .9*forager_survival_rate)[1] more_forager_survival_rate = getPop(1000, egg_laying_rate, egg_fertilization_rate, egg_survival_rate, larva_survival_rate, pupa_survival_rate, drone_survival_rate, worker_survival_rate, 1.1*forager_survival_rate)[1]

The equation $S_d[T_{brood}, r_{egg}]$ gives the daily survival rate of the honeybee eggs given the temperature of the honeycomb cell where the brood is held (T_{brood}), and the baseline egg survival rate (r_{egg}). An estimate for the egg S_d egg is detailed within Section 3.1. An equation for the number of eggs that hatch on a given day can be yielded when taking S_d, when taking into account .

Appendix 10.2: Model Equations

10.2.1 Population at the End of a Given Day

$$P_f = P_i \cdot \prod_{k=0}^{d} S_k$$

10.2.2 Expanded Population Equations

$$P_{stage} = (E_{prev}[d]) + (P_{stage}[d - 1]) - (P_{stage}[d - 1] \cdot (1 - S_d)) - (N[d - v] \cdot \prod_{k=d-v}^{d} S_k)$$

$$P_{stage} = E_{prev}[d] - P_{stage}[d - 1] \cdot S_d - N[d - v] \cdot \prod_{k=d-v}^{d} S_k$$

$$P_{larva} = (l_{egg}[d - v_{egg}] \cdot \prod_{k=d-v_{egg}}^{d} S_k^{egg}) - (P_{larva}[d - 1] \cdot S_d^{larva}) - (N[d - v_{larva}] \cdot \prod_{k=d-v_{larva}}^{d} S_k^{larva})$$

$$P_{pupa} = (E_{larva}[d]) - (P_{pupa}[d - 1] \cdot S_d^{pupa}) - (N[d - v_{pupa}] \cdot \prod_{k=d-v_{pupa}}^{d} S_k^{pupa})$$

$$P_{drone} = (E_{pupa}[d] \cdot (1 - f[d - v_{larva} - v_{pupa}])) - (P_{drone}[d - 1] \cdot S_d^{drone})$$

$$P_{worker} = (E_{pupa}[d] \cdot f[d - v_{larva} - v_{pupa}]) - (P_{worker}[d - 1] \cdot S_d^{worker})$$

$$- (N[d - v_{worker}] \cdot \prod_{k=d-v_{worker}}^{d} S_k^{worker})$$

$$P_{forager} = (E_{worker}[d]) - (P_{drone}[d - 1] \cdot S_d^{drone})$$